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Discrete torsion orbifolds and D-branes

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Abstract: D-branes on orbifolds with and without discrete torsion are analysed in a unified way using the boundary state formalism. For the example of the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold it is found that both the theory with and without discrete torsion possess D-branes whose world-volume carries conventional and projective representations of the orbifold group. The resulting D-brane spectrum is shown to be consistent with T-duality.

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1. Introduction

Much has been learned in the last few years about D-branes in string theory, and this has deepened our understanding of many aspects of the theory. D-branes play a crucial rôle in testing the various duality relations between string theories. D-branes also provide new insights into the background geometry of string theory since the geometry can be analysed in terms of the low-energy theory on the world-volume of a brane probe.

The D-brane spectrum of a number of theories is understood in detail. These include the standard ten-dimensional type-IIA, -IIB and -I theory (see [1] for a review), as well as their non-supersymmetric cousins, type-0A, -0B and -0 [2, 3, 4] (for an earlier non-supersymmetric orientifold construction see also [5]). It is understood how the D-brane spectrum is modified upon compactification on supersymmetric orbifolds and near ALE singularities [6]–[12]. There has also been progress in understanding the D-brane spectrum of Gepner models [13, 14], more general Calabi-Yau manifolds [15]–[23] and WZW models [24]–[29].

Recently, D-branes on orbifolds and orientifolds with discrete torsion [30, 31] have attracted some interest [10], [32]–[39]. The geometry of discrete torsion orbifolds is only partially understood [31, 40, 41, 42], and it is therefore interesting to study what can be learned about it from the analysis of D-brane probes. It was argued in [32] that D-branes in orbifolds with discrete torsion are characterised by the property that the representation of the orbifold group in the corresponding open string description is a *projective* representation. One such brane was analysed in detail, and it was found that its moduli space has a structure that is in agreement with the expectations based on [31, 40]. This analysis was extended in [33] to a more general class of orbifolds.

As was mentioned in [33], one of the models is T-dual to an orbifold without discrete torsion [31]. Since the orbifold with discrete torsion has a brane for which the orbifold group acts projectively on the Chan-Paton factors of the open string, this raises the question of what the T-dual of this brane in the theory without discrete torsion should be [33]. (Similarly, it also raises the question of what the images of the fractional branes of the theory without discrete torsion in the theory with discrete torsion are.) In this paper we propose an answer to both of these questions: we shall argue that *both* orbifolds with and without discrete torsion have D-branes for which the open string has a projective representation of the orbifold group; conversely, *both* orbifolds also have D-branes for which the open string has a proper representation of the orbifold group. Our analysis is based on the construction of boundary states that can be performed irrespective of whether the theory has discrete torsion or not. In fact, since discrete torsion is a relative concept, there does not seem to exist an abstract sense in which a given theory has discrete torsion or not; one should therefore expect that the standard analysis for branes on orbifolds without discrete torsion should equally apply for the case with discrete torsion. The D-brane spectrum we find is consistent with T-duality, it accounts for all the R-R charges of the theory, and it leads to open strings that satisfy the open-closed consistency condition.

The paper is organised as follows. In section 2 we review briefly the main concepts of discrete torsion, and in section 3 we describe a $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold with and without discrete torsion in detail. The boundary states for both orbifolds are constructed in section 4. In particular, we describe in some detail the boundary state description of the D-brane that leads to a projective representation of the orbifold group in the open string. We also explain why T-duality requires that orbifolds with discrete torsion also have branes that lead to conventional representations in the open string, and why, conversely, orbifolds without discrete torsion also have to have branes with projective open string representations. In section 5 we re-examine the consistency argument of Gomis [37] and explain why it is consistent with what we propose. Finally section 6 contains some conclusions. We have included an appendix where the relation between discrete torsion phases and the second cohomology of the orbifold group with coefficients in U(1) is spelled out in some detail.

2. Discrete torsion

Let us briefly recall the definition of discrete torsion in orbifolds. Suppose we consider the orbifold of a closed string theory on $\mathcal{M}$ by the (abelian) group Γ. As is well known [43], the orbifold theory consists of the invariant subspace of the original theory under the action of the orbifold group Γ. In addition, the theory has so-called twisted sectors that describe those closed strings that are only closed in $\mathcal{M}/\Gamma$, but not in $\mathcal{M}$. For an abelian orbifold we have a twisted sector $\mathcal{H}_h$ for each element $h \in \Gamma$. Each twisted sector has to be projected again onto the states that are invariant under the orbifold group Γ; the corresponding projector is of the form

$$P = \frac{1}{|\Gamma|} \sum_{g \in \Gamma} g \, . \tag{2.1}$$

The total partition function of the theory is then

$$Z(q, \bar{q}) = \frac{1}{|\Gamma|} \sum_{g,h \in \Gamma} Z(q, \bar{q}; g, h) \, , \tag{2.2}$$

where

$$Z(q, \bar{q}; g, h) = \mathrm{Tr}_{\mathcal{H}_h} (q^{L_0} \bar{q}^{\bar{L}_0} g) \, . \tag{2.3}$$

The theory with discrete torsion [30] is characterised by the property that the partition function is

$$Z(q, \bar{q}) = \frac{1}{|\Gamma|} \sum_{g,h \in \Gamma} \epsilon(g, h) Z(q, \bar{q}; g, h) \, , \tag{2.4}$$

where $\epsilon(g, h)$ are phases. Modular invariance at one loop requires that

$$\epsilon(g, h) = \epsilon(g^a h^b, g^c h^d) \qquad \text{where} \qquad ad - bc = 1 \qquad \text{and} \qquad a, b, c, d \in \mathbb{Z} \, . \tag{2.5}$$

Furthermore modular invariance on higher genus surfaces, together with the factorization property of loop amplitudes, implies that the $\epsilon(g, h)$ have to define a one-dimensional representation of Γ,

$$\epsilon(g_1 g_2, h) = \epsilon(g_1, h) \epsilon(g_2, h) \, . \tag{2.6}$$

The set of inequivalent different torsion theories are classified by the second cohomology group of Γ with values in $\mathrm{U}(1)$, $H^2(\Gamma, \mathrm{U}(1))$.[1] This cohomology group consists of the two-cocycles $c(g, h) \in \mathrm{U}(1)$ satisfying the cocycle condition

$$c(g_1, g_2 g_3) c(g_2, g_3) = c(g_1 g_2, g_3) c(g_1, g_2) \, , \tag{2.7}$$

where we identify cocycles that differ by a coboundary,

$$c'(g, h) = \frac{c_g c_h}{c_{gh}} c(g, h) \, . \tag{2.8}$$

[1]The following discussion follows closely [37].

(Here $c_g \in \mathrm{U}(1)$ for each $g \in \Gamma$.) Indeed, for each such cocycle one can define

$$\epsilon(g,h) = \frac{c(g,h)}{c(h,g)} \,. \tag{2.9}$$

It follows immediately from this definition that

$$\epsilon(g,g) = 1 \qquad \epsilon(g,h) = \epsilon(h,g)^{-1} \,, \tag{2.10}$$

and a short calculation shows that (2.7) implies (2.6). Together with (2.10) this is then sufficient to prove (2.5) (see [30]). It is also manifest from (2.9) that the definition of $\epsilon(g,h)$ is the same for cocycles that differ by a coboundary. Conversely, one can construct a cocycle c for each consistent set of discrete torsion phases so that (2.9) is satisfied; this is described in detail in the appendix.[2]

Because of the relation between (2.2) and (2.1), the modification of the partition function implies that the projection operator onto physical states in the sector $\mathcal{H}_h$ is modified to be

$$P|_{\mathcal{H}_h} = \frac{1}{|\Gamma|} \sum_{g\in\Gamma} \epsilon(g,h) g|_{\mathcal{H}_h} \,. \tag{2.11}$$

In particular, this implies that a physical state in the twisted sector $\mathcal{H}_h$ satisfies

$$g|s\rangle_{\mathcal{H}_h} = \epsilon(g,h)^* |s\rangle_{\mathcal{H}_h} \,. \tag{2.12}$$

Alternatively, one can interpret the theory with discrete torsion as the theory where $g \in \Gamma$ acts on the $\mathcal{H}_h$ sector as

$$\hat{g}|_{\mathcal{H}_h} = \epsilon(g,h) g|_{\mathcal{H}_h} \,. \tag{2.13}$$

Because of (2.6) this gives a well-defined action of Γ on each $\mathcal{H}_h$. From the point of view of conformal field theory, the orbifold with discrete torsion can therefore be thought of as a standard orbifold (without discrete torsion) where the elements of Γ act as $\hat{g}$ on the various sectors. In particular, this implies that (at least from this perspective) there is no abstract sense in which one can say that a given orbifold is an orbifold with discrete torsion; rather, discrete torsion is a relative concept that describes how to obtain a consistent orbifold from another consistent orbifold (in a way that does not modify the action of the orbifold group in the untwisted sector). One should therefore expect that D-branes on orbifolds "with torsion" can be discussed and described using the same techniques as in the case "without torsion". This is indeed what we shall find.

[2] Naively one may think that $c(g,h)$ can simply be defined by $c(g,h) = \epsilon(g,h)^{1/2}$ since (2.5) and (2.6) imply that $\epsilon(g,h) = \epsilon(h,g)^{-1}$, and therefore (2.9) reproduces $\epsilon(g,h)$. In addition (2.6) implies that the square of the left-hand side in (2.7) equals the square of the right-hand side, but this only implies that (2.7) holds up to sign. In fact, there does not seem to exist a "natural" sign conventions for the definition of $c(g,h) = \epsilon(g,h)^{1/2}$ that lead to $c(g,h)$ satisfying (2.7). In appendix A we shall therefore follow a different route.

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3. An example of an orbifold with discrete torsion

The simplest example of an orbifold where discrete torsion is possible is the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold of type II on T^6, where the generators of the orbifold group, g_1 and g_2 act as in table 1. Here we have defined $g_3 = g_1 g_2$; a $+$ sign in the above table means that g_i acts as the identity on the corresponding coordinate, whereas a $-$ sign indicates that g_i acts as $x^j \mapsto -x^j$.

	x^3	x^4	x^5	x^6	x^7	x^8
g_1	$+$	$+$	$-$	$-$	$-$	$-$
g_2	$-$	$-$	$+$	$+$	$-$	$-$
g_3	$-$	$-$	$-$	$-$	$+$	$+$

Table 1: The action of the orbifold group.

In this case, there are two possible choices for the signs $\epsilon(g,h)$: either we choose $\epsilon(g,h) = +1$ for all $g,h \in \Gamma \cong \mathbb{Z}_2 \times \mathbb{Z}_2$, or we define

$$\epsilon(g,h) = \begin{cases} +1 & \text{if } g = e,\ h = e \text{ or } g = h \\ -1 & \text{otherwise}\,. \end{cases} \tag{3.1}$$

The first solution corresponds to the trivial cocycle $c(g,h) = +1$ for all $g,h \in \Gamma$, whereas (3.1) comes from

$$c(g,h) = \begin{cases} +1 & \text{if } g = e \text{ or } h = e \text{ or } g = g_2 \text{ or } h = g_1 \\ -1 & \text{otherwise}\,. \end{cases} \tag{3.2}$$

In either case, the resulting manifold is a Calabi-Yau manifold [31], and the theory therefore preserves supersymmetry. This can be used to determine the relevant GSO-projections in the various sectors. For the case of the type-IIA orbifold, the correct GSO-projection is

$$\textbf{IIA} \qquad \begin{array}{ll} \frac{1}{4}(1+(-1)^F)(1+(-1)^{\tilde{F}}) & \text{in all NS-NS sectors} \\ \frac{1}{4}(1+(-1)^F)(1-(-1)^{\tilde{F}}) & \text{in all R-R sectors}\,, \end{array} \tag{3.3}$$

while for the case of the type-IIB orbifold we have

$$\textbf{IIB} \qquad \begin{array}{ll} \frac{1}{4}(1+(-1)^F)(1+(-1)^{\tilde{F}}) & \text{in all NS-NS sectors} \\ \frac{1}{4}\left(1+(-1)^F\right)\left(1+(-1)^{\tilde{F}}\right) & \text{in all R-R sectors}\,. \end{array} \tag{3.4}$$

Alternatively, this can be determined using the approach of [44] (see further below). The GSO-projection is the same for either choice of ϵ.

The definition of g_i in all sectors that have fermionic zero modes is *a priori* ambiguous; we choose the convention that on the ground states g_1 acts as[3]

$$g_1 = \prod_{i \in \mathcal{Z}} \left(\sqrt{2}\psi_0^i\right) \prod_{i \in \mathcal{Z}} \left(\sqrt{2}\widetilde{\psi}_0^i\right), \tag{3.5}$$

[3]The fermionic zero modes are normalised so that $\{\psi_0^\mu, \psi_0^\nu\} = \delta^{\mu,\nu}$ and similarly for $\widetilde{\psi}_0^\mu$.

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where $\mathcal{Z} \subset \{5, 6, 7, 8\}$ is the set of coordinates along which there are fermionic zero modes, and the ordering of the zero modes in the two products is the same. The formulae for g_2 and g_3 are analogous. There exists another natural definition for g_i, where g_1 acts on the ground states as

$$\hat{g}_1 = \prod_{i \in \mathcal{Z}} \left(2\psi_0^i \widetilde{\psi}_0^i \right), \tag{3.6}$$

and $\hat{g}_2$ and $\hat{g}_3$ are analogously defined. These two definitions are precisely related by discrete torsion, namely

$$\hat{g}_i|_{\mathcal{H}_{g_j}} = \epsilon(g_i, g_j) g_i|_{\mathcal{H}_{g_j}}, \tag{3.7}$$

where ϵ is defined as in (3.1). In this example it is clear that it is a matter of convention which of the two theories one interprets as the orbifold with torsion, and which as the orbifold without torsion.

However, there is one statement that can be made irrespective of this convention: the T-dual of the IIA theory without torsion (where we T-dualise along x^3, x^5 and x^7, say) is the IIB theory with torsion and vice versa [31]. In fact, the theory where g_i acts as in (3.5) has the Hodge diamond

$$\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 3 & 51 & 0 \\ 0 & 51 & 3 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}, \tag{3.8}$$

while for the theory where g_i acts as in (3.6) the Hodge diamond is

$$\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 51 & 3 & 0 \\ 0 & 3 & 51 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}. \tag{3.9}$$

This suggests that the two theories are actually mirror partners of each other which is indeed the case [31]. Since we know how D-branes behave under T-duality, this example provides consistency conditions on the D-brane spectrum of theories with and without discrete torsion. We shall check later that our description of the D-brane spectrum satisfies this constraint.

4. Boundary states and D-branes

One method for the analysis of D-branes is the boundary state approach in which D-branes are described in terms of coherent (boundary) states of the underlying closed string theory [45, 46, 2] (see also [47, 48, 49] for reviews). For the case of the above orbifold (without discrete torsion), this analysis has been performed in [22],

	$(-1)^{\tilde{F}}$	g_1	g_2	g_3
NS-NS;U	$+1$	$+1$	$+1$	$+1$
R-R;U	$(-1)^{r+s_1+s_2+s_3+1}$	$(-1)^{s_2+s_3}$	$(-1)^{s_1+s_3}$	$(-1)^{s_1+s_2}$
NS-NS;T$_{g_1}$	$(-1)^{s_2+s_3}$	$(-1)^{s_2+s_3}$	$(-1)^{s_3}$	$(-1)^{s_2}$
R-R;T$_{g_1}$	$(-1)^{r+s_1+1}$	$+1$	$(-1)^{s_1}$	$(-1)^{s_1}$
NS-NS;T$_{g_2}$	$(-1)^{s_1+s_3}$	$(-1)^{s_3}$	$(-1)^{s_1+s_3}$	$(-1)^{s_1}$
R-R;T$_{g_2}$	$(-1)^{r+s_2+1}$	$(-1)^{s_2}$	$+1$	$(-1)^{s_2}$
NS-NS;T$_{g_3}$	$(-1)^{s_1+s_2}$	$(-1)^{s_2}$	$(-1)^{s_1}$	$(-1)^{s_1+s_2}$
R-R;T$_{g_3}$	$(-1)^{r+s_3+1}$	$(-1)^{s_3}$	$(-1)^{s_3}$	$+1$

Table 2: The transformation properties of the boundary states $(r; s_1, s_2, s_3)$.

and we collect the relevant results here. Following [12, 22], we denote a Dirichlet p-brane as $(r; s_1, s_2, s_3)$ where $p = r+s_1+s_2+s_3$, provided that it has $r+1$ Neumann boundary conditions along the directions that have not been affected by the orbifold, i.e. x^0, x^1, x^2, x^9, and s_i Neumann boundary conditions along the directions x^{2i+1} and x^{2i+2}.

As is familiar in theories with world-sheet fermions, the actual boundary state is a linear combination of boundary states for different values of the parameter η labelling the different spin structures. In each sector of the theory, there is a (up to normalisation) unique linear combination that is invariant under $(-1)^F$; under the action of the various other operators (where the action of g_i is defined as in (3.5)), this state transforms as in table 2.

4.1 The theory without discrete torsion

Having collected this information, we can now describe the D-brane spectrum of these orbifold theories. Let us first consider the case "without torsion" (i.e. the theory where g_i acts as (3.5), rather than as (3.6) in the twisted sectors). We shall mainly concentrate on the BPS branes, although these theories also contain interesting (stable) non-BPS branes [50, 51, 22]. First of all the orbifold theory contains the familiar "fractional branes"; these branes are localised at one of the 64 fixed planes of Γ, and their open string spectrum (of the open string that begins and ends on the same brane) is of the form

$$\frac{1}{8}(1+(-1)^F)(1+g_1)(1+g_2)\,. \tag{4.1}$$

The corresponding boundary state has a non-trivial component in all untwisted and twisted sectors (where the twisted sectors are all associated to the same fixed plane). In order for this to be possible, we have the following restriction on $(r; s_1, s_2, s_3)$:

$$\text{Fractional D-branes} \qquad \begin{array}{ll} \text{IIA:} & r \text{ even and } s_i \text{ even}\,, \\ \text{IIB:} & r \text{ odd and } s_i \text{ even}\,. \end{array} \tag{4.2}$$

Unlike what happens in simpler orbifold theories (such as the ones studied in [51, 12]), the lattice of D-brane charges is not generated by these fractional branes alone. Indeed, it is manifest from table 2 that there is a boundary state in the untwisted R-R sector that is invariant under all projection operators provided that all s_i are odd (i.e. equal to 1) and that r is odd for IIA and even for IIB. One can therefore construct an additional "bulk" D-brane, i.e. a boundary state with only untwisted components [22]. However this bulk brane is *not* the minimally charged object. Indeed, one can construct an "almost fractional" boundary state whose charge is half the charge of the bulk brane. The moduli space of the corresponding brane[4] consists of the different fixed planes of g_1, g_2 or g_3. The boundary state description of the brane is slightly different for the different branches of the moduli space, and we shall in the following only give the explicit formula for the brane that stretches between the fixed planes of g_1 defined by $x^6 = x^8 = 0$, where x^5 and x^7 take the values 0, πR^5 and 0, πR^7, respectively; the formulae for the other branches (and orientations) are analogous.

Let us denote by $\mathbf{y}$ the position of the brane in the directions that are unaffected by the orbifold action, by $\mathbf{a}$ the coordinates in the x^3, x^4 directions on the fixed planes of g_1, and by $\mathbf{b_i}$, $i = 1, 2, 3, 4$ the coordinates (in the x^5, x^6, x^7, x^8 directions) of the four fixed planes. (So, for example, $\mathbf{b_1} = (0, 0, 0, 0)$, $\mathbf{b_2} = (\pi R^5, 0, 0, 0)$, $\mathbf{b_3} = (0, 0, \pi R^7, 0)$, $\mathbf{b_4} = (\pi R^5, 0, \pi R^7, 0)$.) The relevant boundary state is then of the form

$$
\begin{aligned}
|D(r;1,1,1);\mathbf{y},\mathbf{a},\epsilon\rangle = {} & |D(r;1,1,1);\mathbf{y},\mathbf{a}\rangle_{\mathrm{NS-NS;U}} + \epsilon|D(r;1,1,1);\mathbf{y},\mathbf{a}\rangle_{\mathrm{R-R;U}} + \\
& + \sum_{i=1}^{4}\Big(|D(r;1,1,1);\mathbf{y},\mathbf{a}\rangle_{\mathrm{NS-NS;T}_{g_1,\mathbf{b_i}}} + \\
& \qquad + \epsilon|D(r;1,1,1);\mathbf{y},\mathbf{a}\rangle_{\mathrm{R-R;T}_{g_1,\mathbf{b_i}}}\Big) + \\
& + |D(r;1,1,1);\mathbf{y},-\mathbf{a}\rangle_{\mathrm{NS-NS;U}} + \epsilon|D(r;1,1,1);\mathbf{y},-\mathbf{a}\rangle_{\mathrm{R-R;U}} - \\
& - \sum_{i=1}^{4}\Big(|D(r;1,1,1);\mathbf{y},-\mathbf{a}\rangle_{\mathrm{NS-NS;T}_{g_1,\mathbf{b_i}}} + \\
& \qquad + \epsilon|D(r;1,1,1);\mathbf{y},-\mathbf{a}\rangle_{\mathrm{R-R;T}_{g_1,\mathbf{b_i}}}\Big)\,,
\end{aligned} \tag{4.3}
$$

where r is odd for type IIA and even for type IIB, and $\epsilon = \pm 1$ distinguishes between the brane and the antibrane. For simplicity, we have here described the brane without Wilson line along x^5 and x^7 for which the signs of the contributions of the four twisted sectors is the same. It follows from the results of table 2, together with the observation that both g_2 and g_3 act on $\mathbf{a}$ as $\mathbf{a} \mapsto -\mathbf{a}$ that the boundary state in (4.3) is invariant under the action of the whole orbifold group. The brane that is described by (4.3) can be thought to consist of a brane at $(\mathbf{y}, \mathbf{a})$ together with an anti-brane at $(\mathbf{y}, -\mathbf{a})$ (where the untwisted R-R charges of the two branes are the same, whereas

[4] We propose to call branes of this type *projective fractional* D-branes.

the charges with respect to the g_1-twisted sectors are opposite). The open string that corresponds to this boundary state has therefore a 2×2 Chan-Paton matrix

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}, \tag{4.4}$$

where a (d) labels the string that begins and ends at the brane at $\mathbf{a}$ ($-\mathbf{a}$), whereas b denotes the string that begins at $\mathbf{a}$ and ends at $-\mathbf{a}$, and c denotes the same string with the opposite orientation. In terms of the open string, the orbifold generators act on the Chan-Paton matrix by conjugation,

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \longmapsto \gamma(g_i) \begin{pmatrix} a & b \\ c & d \end{pmatrix} \gamma(g_i)^{-1}. \tag{4.5}$$

We can read off from (4.3) how the Chan-Paton matrix transforms under the three orbifold actions, and we find that

$$\begin{aligned} g_1 : \quad & \begin{pmatrix} a & b \\ c & d \end{pmatrix} \longmapsto \begin{pmatrix} a & -b \\ -c & d \end{pmatrix}, \\ g_2 : \quad & \begin{pmatrix} a & b \\ c & d \end{pmatrix} \longmapsto \begin{pmatrix} d & \pm c \\ \pm b & a \end{pmatrix}, \\ g_3 : \quad & \begin{pmatrix} a & b \\ c & d \end{pmatrix} \longmapsto \begin{pmatrix} d & \mp c \\ \mp b & a \end{pmatrix}. \end{aligned} \tag{4.6}$$

For example, g_1 acts with a minus sign on the strings that run between the brane and the anti-brane, and g_2 and g_3 exchange the brane and the anti-brane. On the level of this discussion it is impossible to fix the signs on the off-diagonal elements in the action of g_2 and g_3, but consistency with the group relations, in particular $g_2 g_2 = e$, $g_3 g_3 = e$ and $g_3 = g_1 g_2$, determines the relative signs as above.

The matrices $\gamma(g_i)$ that implement the transformations described in (4.6) by conjugation as in (4.5) are

$$\begin{aligned} \gamma(g_1) &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \\ \gamma(g_2) &= \begin{pmatrix} 0 & \pm 1 \\ 1 & 0 \end{pmatrix}, \\ \gamma(g_3) &= \begin{pmatrix} 0 & \mp 1 \\ 1 & 0 \end{pmatrix}. \end{aligned} \tag{4.7}$$

These matrices define a *projective* representation of $\mathbb{Z}_2 \times \mathbb{Z}_2$; indeed, if we consider the upper sign in (4.7) we have

$$\gamma(g_i)\gamma(g_j) = c(g_i, g_j)\gamma(g_i g_j), \tag{4.8}$$

where c is the cocycle that was defined in (3.2).[5] The existence of a brane with such properties was predicted in [32, 33]; their argument was based on the observation that

[5] The lower sign in (4.7) leads to a cocycle c' that differs from c by the coboundary $c_e = c_{g_1} = 1$, $c_{g_2} = c_{g_3} = +i$.

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the theory with discrete torsion has a brane that leads to a projective representation of the orbifold group, and that T-duality along x^3, x^5, x^7 maps the orbifold theory with discrete torsion to the one without discrete torsion [31].

4.2 The theory with discrete torsion

For the theory with discrete torsion, i.e. the theory where g_i acts as $\hat{g}_i$ on the twisted sectors, the analysis is completely analogous. Fractional branes exist now for

$$\text{Fractional D-branes} \qquad \begin{array}{ll} \mathbf{IIA}: & r \text{ odd and } s_i \text{ odd}\,, \\ \mathbf{IIB}: & r \text{ even and } s_i \text{ odd}\,. \end{array} \tag{4.9}$$

In addition, there are projective fractional branes (whose boundary state is given by a similar expression as in (4.3)) for

$$\text{Projective fractional D-branes} \qquad \begin{array}{ll} \mathbf{IIA}: & r \text{ even and } s_i \text{ even}\,, \\ \mathbf{IIB}: & r \text{ odd and } s_i \text{ even}\,. \end{array} \tag{4.10}$$

The fractional branes are the states in the theory that are charged under the massless fields from the twisted R-R sectors; if these branes did not exist, the theory would not possess *any* states that are charged under these fields.

The projective fractional D-branes were first discussed (in the uncompactified theory) in [32, 33], where it was also observed that their moduli space has three branches (at each of the fixed planes) as we have found above. Under T-duality along x^3, x^5 and x^7, say, the IIA (IIB) theory without discrete torsion is mapped to the IIB (IIA) with discrete torsion and vice versa. On D-branes, this T-duality transformation leaves r invariant, and changes each s_i by ± 1. This is consistent with the D-brane spectrum of the two theories that we have found above.

5. Open-closed consistency condition

It was shown by Gomis [37] that the representations of the orbifold group that appear in the open string description are constrained in terms of the actual representation of the orbifold group on the various twisted sectors of the closed string theory; this is a consequence of the open-closed consistency condition that was first considered, in a slightly different context, in [52]. Superficially, the analysis of Gomis seems to imply that for orbifolds with discrete torsion *only* projective representations of the orbifold group can occur in the open string; as we have seen above, this would be in conflict with T-duality (and our discussion of Dirichlet branes in these theories). We shall now explain that his argument, correctly interpreted, is in precise accord with what we have found above.

Following [37], let us consider the disk diagram where we insert a closed string state in the g_i-twisted sector at the centre of the disk, and an open string vertex operator at the boundary. As is explained in [37], this amplitude is proportional to

$$\mathrm{Tr}(\gamma(g_i)\lambda)\, \langle V(\boldsymbol{\phi}, 0) V(\psi, 1)\rangle\,, \tag{5.1}$$

where λ denotes the Chan-Paton matrix of the open string field ψ, and $\boldsymbol{\phi}$ is the state in the g_i-twisted sector. Here $\gamma(g_i)$ arises because the state in the g_i-twisted sector generates a branch cut from the centre of the disk to the boundary along which fields jump by the action of g_i.

The consistency condition on the allowed representations in the open string arises from the constraint that this amplitude must be invariant under the action of the orbifold group. As we have argued before, the action of the orbifold group on the closed string twisted sectors is given by $\hat{g}$ as in (2.13), where the action of g on the twisted sectors is defined in some natural way, and ϵ describes the relative discrete torsion with respect to the reference theory.[6] For general ϵ, the condition that the amplitude (5.1) is invariant under the action of the orbifold group therefore becomes

$$\begin{aligned}\mathrm{Tr}(\gamma(g_i)\lambda)\,\langle V(\boldsymbol{\phi},0)V(\psi,1)\rangle &= \mathrm{Tr}\left(\gamma(g_i)\gamma(g_j)\lambda\gamma(g_j)^{-1}\right)\,\langle V(\hat{g}_j\boldsymbol{\phi},0)V(g_j\psi,1)\rangle \\ &= \epsilon(g_j,g_i)\,\mathrm{Tr}\left(\gamma(g_i)\gamma(g_j)\lambda\gamma(g_j)^{-1}\right)\times \\ &\quad\times\langle V(g_j\boldsymbol{\phi},0)V(g_j\psi,1)\rangle\,, \end{aligned} \tag{5.2}$$

where we have used that the action on the open string Chan-Paton indices is defined by (4.5).

Let us consider an open string state ψ that is invariant under the action of the orbifold group, $g_j\psi=\psi$, and let us denote by $\delta_j(\boldsymbol{\phi})$ the eigenvalue of $\boldsymbol{\phi}$ under the action of g_j, $g_j\boldsymbol{\phi}=\delta_j(\boldsymbol{\phi})\boldsymbol{\phi}$. If $\boldsymbol{\phi}$ is a physical state in the orbifold theory with relative discrete torsion ϵ, then we have to have $\delta_j(\boldsymbol{\phi})\epsilon(g_j,g_i)=+1$. If in addition the amplitude $\langle V(\boldsymbol{\phi},0)V(\psi,1)\rangle$ does not vanish, then the consistency condition implies that

$$\mathrm{Tr}(\gamma(g_i)\lambda)=\mathrm{Tr}\left(\gamma(g_i)\gamma(g_j)\lambda\gamma(g_j)^{-1}\right). \tag{5.3}$$

The Chan-Paton matrix λ is arbitrary, and this statement is therefore equivalent to

$$\gamma(g_j)\gamma(g_i)=\gamma(g_i)\gamma(g_j)\,. \tag{5.4}$$

In particular, it then follows that the representation of the orbifold group defined by γ is a proper (not a projective) representation. This conclusion applies to those open strings that have a non-vanishing coupling with physical states in the twisted sector of the orbifold theory. This is in particular the case for the fractional D-branes we have discussed above.

On the other hand, the situation that was considered by Gomis corresponds to the case when $\delta_j(\boldsymbol{\phi})=+1$, i.e. when the open string state couples to a closed string

[6] This is where our analysis differs from that of Gomis: he assumes that the action of the orbifold group on the twisted sectors is unmodified for the case of discrete torsion, and that only the condition on physical states is modified as in (2.12). These two points of view are equivalent for the closed string sector of the theory, but they lead to different conclusions once open strings are considered as well. As we will see, our point of view reproduces the above D-brane spectrum that is consistent with T-duality.

state that is physical in the theory without discrete torsion, but unphysical provided that $\epsilon(g_i, g_j) \neq 1$. Again under the assumption that the corresponding overlap does not vanish, an analogous argument then implies that

$$\gamma(g_j)\gamma(g_i) = \epsilon(g_j, g_i)\gamma(g_i)\gamma(g_j)\,. \tag{5.5}$$

If we rewrite $\epsilon(g_j, g_i)$ in terms of the cocylce c as in (2.9), this becomes

$$\gamma(g_j)\gamma(g_i)c(g_i, g_j) = c(g_j, g_i)\ \gamma(g_i)\gamma(g_j)\,. \tag{5.6}$$

The representation of the orbifold group in the open string is then the projective representation described by

$$\gamma(g_i)\gamma(g_j) = c(g_i, g_j)\gamma(g_i g_j)\,. \tag{5.7}$$

This analysis applies to the projective fractional branes for which the twisted closed string states to which the D-brane would normally couple are unphysical.

From this point of view, the question of whether the representation of the orbifold group on the open string Chan-Paton indices is a proper representation or a projective representation does *not* depend on whether the theory in question is an orbifold with or without discrete torsion; it only depends on the transformation properties of the twisted sector states to which the open string state couples.

6. Conclusions

In this paper we have re-examined the D-brane spectrum of a certain $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold with and without discrete torsion. We have argued, on general grounds, that the analysis for the two cases must be analogous, and we have shown that this leads to a D-brane spectrum that is consistent with T-duality that relates the theory with and without discrete torsion [31]. The picture that seems to be emerging is that the representation of the orbifold group in the open string description can either be a proper or a projective representation, irrespective of whether the orbifold theory has discrete torsion or not. The emergence of projective representations of the orbifold group is merely related to discrete torsion in the sense that only orbifold theories that admit discrete torsion also admit projective representations that are not equivalent to proper representations.[7] This conclusion is somewhat different from what was argued for in [32, 33, 37]. We have also given a boundary state description for branes that lead to projective open string representations, and we have found that their moduli space has the same structure as perdicted in [32, 33].

In this paper we have only analysed the $\mathbb{Z}_2 \times \mathbb{Z}_2$ case that is special in that T-duality relates the theory with discrete torsion to the one without. It would be

[7] Every such projective representation gives rise to a non-trivial co-cycle in $H^2(\Gamma, \mathrm{U}(1))$.

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interesting to see how the findings of this paper generalise for more general orbifolds with (and without) discrete torsion; this is currently under consideration [53]. It would also be interesting to understand whether non-BPS D-branes can also have projective representations of the orbifold group, and how this fits together with the various decay processes of non-BPS branes into brane anti-brane pairs.

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A. The construction of the two-cocycle

Let us first determine the most general set of phases $\epsilon(g,h)$ that satisfy (2.5) and (2.6). Recall that every finitely generated abelian group Γ can be written as (see for example [30])

$$\Gamma = \mathbb{Z}_{m_1} \times \mathbb{Z}_{m_2} \times \cdots \times \mathbb{Z}_{m_k} \,, \tag{A.1}$$

where m_i is a factor of m_{i+1}. Let us denote by α_i the generator of $\mathbb{Z}_{m_i}$. Then every set of discrete torsion phases is uniquely determined by the set of phases

$$\epsilon_{ij} = \epsilon(\alpha_i, \alpha_j) \,, \qquad \text{where } i < j \,. \tag{A.2}$$

Indeed, it is easy to check that (2.5) and (2.6) imply (2.10), and the second equation in (2.10) determines then ϵ on all pairs of generators. Using the representation property (2.6) this finally fixes ϵ for all pairs (g,h). Provided that ϵ_{ij} is a m_ith root of unity, this construction is well defined. Since the resulting phases satisfy by construction (2.6) and (2.10), it follows by the same arguments as in the main part of the paper that they also satisfy (2.5). We have therefore shown that the set of possible discrete torsion phases is given by

$$\mathbb{Z}_{m_1}^{k-1} \times \mathbb{Z}_{m_2}^{k-2} \times \cdots \times \mathbb{Z}_{m_{k-1}} \,. \tag{A.3}$$

The set of possible discrete torsion phases is actually a group since the product of two sets of discrete torsion phases defines another set of discrete torsion phases. This group is generated by the *primitive* discrete torsion phases for which $\epsilon_{ij} = 1$ for all but one pair $i < j$ for which ϵ_{ij} is a primitive m_ith root of unity.

The set of cocylces is also an abelian group (where the group multiplication is also given by pointwise multiplication). In order to construct a cocycle c for each set of discrete torsion phases ϵ (so that ϵ is determined in terms of c by (2.9)), it is therefore sufficient to construct such a cocycle for the primitive discrete torsion phases only. This can be done indirectly, by constructing a certain projective representation of Γ [33].

In order to simplify notation, let us consider the case where the primite discrete torsion phases are defined by $\epsilon_{12} \neq 1$, with $\epsilon \equiv \epsilon_{12}$ a primitive $m \equiv m_1$th root of unity. There are two cases to consider: if m is odd, we construct a m-dimensional projective representation by

$$\gamma(\alpha_1) = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 0 & 1 \\ 1 & 0 & 0 & \cdots & 0 \end{pmatrix},$$
$$\gamma(\alpha_2) = \begin{pmatrix} 0 & \epsilon & 0 & \cdots & 0 \\ 0 & 0 & \epsilon^2 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 0 & \epsilon^{m-1} \\ 1 & 0 & 0 & \cdots & 0 \end{pmatrix}, \tag{A.4}$$

together with $\gamma(\alpha_j) = \mathbf{1}_m$. On the other hand, if m is even, $\gamma(\alpha_j)$ for $j \neq 2$ is as above, and we define

$$\gamma(\alpha_2) = \begin{pmatrix} 0 & \delta & 0 & \cdots & 0 \\ 0 & 0 & \delta^3 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 0 & \delta^{2m-3} \\ \delta^{2m-1} & 0 & 0 & \cdots & 0 \end{pmatrix}, \qquad (m \text{ even}), \tag{A.5}$$

where $\delta^2 = \epsilon$. This construction guarantees that $\gamma(\alpha_j)^m = \mathbf{1}$ for all j. Furthermore we have

$$\gamma(\alpha_1)\gamma(\alpha_2) = \epsilon\gamma(\alpha_2)\gamma(\alpha_1), \tag{A.6}$$

while all other generators commute pairwise.

Next we extend this to a (projective) representation of Γ by defining

$$\gamma(g) = \prod_j \gamma\left(\alpha_{i_j}\right), \tag{A.7}$$

where we choose for each element $g \in \Gamma$ a realisation as $g = \prod_j \alpha_{i_j}$, and we pick a specific order for the $\gamma(\alpha_{i_j})$ on the right-hand side of (A.7). Because of the commutation relations and the property that $\gamma(\alpha_j)^m = \mathbf{1}$ for all j, we then find that

$$\gamma(g)\gamma(h) = c(g,h)\gamma(gh), \tag{A.8}$$

where $c(g,h)$ are certain phases. These phases satisfy the cocycle condition (2.7) since the representation of Γ is associative.

It follows directly from (A.6) that $\epsilon(g,h)$, defined by (2.9), satisfies $\epsilon(\alpha_i,\alpha_j)=1$ unless $(i,j)=(1,2)$ or $(i,j)=(2,1)$, and that $\epsilon(\alpha_1,\alpha_2)=\epsilon$. Since each cocylce defines a set of phases that satisfy (2.5) and (2.6), it therefore follows that the above cocycle reproduces indeed the desired discrete torsion phases.

Finally, in order to prove that the correspondence between discrete torsion phases and cocycles is one-to-one, it remains to check that the map defined by (2.9) is injective. Let us therefore assume that for a given cocycle $c(g,h)$, all $\epsilon(g,h)=+1$, i.e. that $c(g,h)=c(h,g)$ for all $g,h\in\Gamma$. We want to show that we can find a coboundary so that c', as defined in (2.8), satisfies $c'(g,h)=1$ for all $g,h\in\Gamma$.

Without loss of generality (by choosing $c_e=c(e,e)^{-1}$) we may assume that $c(e,e)=1$. The cocycle condition (2.7) then implies that $c(e,g)=c(g,e)=1$ for all $g\in\Gamma$. Let us then define c_g for all $g\in\Gamma$ in terms of c_{α_i} by the formula

$$\begin{aligned} c_{\alpha_{i_1}\cdots\alpha_{i_l}} &= c(\alpha_{i_1}\cdots\alpha_{i_{l-1}},\alpha_{i_l})\,c(\alpha_{i_1}\cdots\alpha_{i_{l-2}},\alpha_{i_{l-1}})\cdots c(\alpha_{i_1}\alpha_{i_2},\alpha_{i_3})\,c(\alpha_{i_1},\alpha_{i_2})\times \\ &\quad\times\prod_{j=1}^{l} c_{\alpha_{i_j}}\,. \end{aligned} \tag{A.9}$$

Using the cocycle condition (2.7) and the property that $c(g,h)=c(h,g)$ it is easy to see that this definition is independent of the order of the α_i. In order to show that it respects the group relations, we observe that

$$c_{\alpha_{i_1}\cdots\alpha_{i_l}\alpha_{j_1}\cdots\alpha_{j_m}} = c\left(\alpha_{i_1}\cdots\alpha_{i_l},\alpha_{j_1}\cdots\alpha_{j_m}\right)c_{\alpha_{i_1}\cdots\alpha_{i_l}}c_{\alpha_{j_1}\cdots\alpha_{j_m}}\,, \tag{A.10}$$

where we have again used the cocycle condition. We can choose c_{α_i} so that $c_{\alpha_i\cdots\alpha_i}$, as defined by the right-hand side of (A.9), satisfies

$$c_{\underbrace{\alpha_i\cdots\alpha_i}_{m_i}} = c_e = 1\,. \tag{A.11}$$

Because of (A.10) and the fact that $c(e,g)=c(g,e)=1$, this then implies that (A.9) respects the group relations. Finally, it is manifest from (A.10) that $c'(g,h)$, defined by (2.8), satisfies then $c'(g,h)=1$ for all $g,h\in\Gamma$.

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